Worked Solutions

Edexcel C3 Paper L

1. $\dfrac{x}{(x-3)(x+3)} - \dfrac{1}{(x-3)(x-1)}$

$\dfrac{x(x-1)-(x+3)}{(x-3)(x+3)(x-1)}$

$\dfrac{x^2-2x-3}{(x-3)(x+3)(x-1)}$

$\dfrac{(x-3)(x+1)}{(x-3)(x+3)(x-1)} = \dfrac{x+1}{(x+3)(x-1)}$ **(6)**

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2. (*a*) $\mathrm{f}(0) = \mathrm{e}^3 \quad \mathrm{ff}(0) = \mathrm{e}^{2\mathrm{e}^3+3}$ **(2)**

(*b*) let $y = \mathrm{e}^{2x+3}$

$\ln y = 2x + 3$

$\dfrac{1}{2}(\ln y - 3) = x \quad \therefore \mathrm{f}^{-1}(x) = \dfrac{1}{2}(\ln x - 3)$ **(3)**

(*c*) domain for $\mathrm{f}^{-1}(x)$ is $x \in \mathbb{R},\ x > 0$ **(1)**

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3. $\dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{2y}{y^2+4} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y^2+4}{2y} = \dfrac{y^2}{2y} + \dfrac{4}{2y}$ **(6)**

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4. (*a*) $\left.\begin{array}{l}\mathrm{f}(1) = \ln 1 - 3 + 5 = 2 \\ \mathrm{f}(2) = \ln 2 - 6 + 5 = -0.306\end{array}\right\}$ change of sign $\therefore$ root in interval **(2)**

(*b*) $x_1 = 1.8977, x_2 = 1.8802, x_3 = 1.8771, x_4 = 1.8766$ **(3)**

(*c*) $\left.\begin{array}{l}\mathrm{f}(1.8765) = -9.666 \times 10^{-5} \\ \mathrm{f}(1.8755) = 2.375 \times 10^{-3}\end{array}\right\}$ sign change

$\therefore \alpha = 1.876$ (4 sig. fig.) **(2)**

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5. (*a*) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sec^2 x + 2\cos 2x$

at $x = \dfrac{\pi}{4}$,

$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(\sqrt{2}\right)^2 + 0 = 2$

(b) at $x = \dfrac{\pi}{4}$,

$y = 1 + 1 = 2$

equation of tangent is $y - 2 = 2\left(x - \dfrac{\pi}{4}\right)$

$y = 2x + 2 - \dfrac{\pi}{2}$

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6. (*a*) R.H.S $= \dfrac{2\tan\theta}{\sec^2\theta}$

$= 2 \cdot \dfrac{\sin\theta}{\cos\theta} \cdot \cos^2\theta$

$= 2\sin\theta\cos\theta$

$= \sin 2\theta$

(*b*) $4\tan\theta - \tan^2\theta = 1$

$4\tan\theta = 1 + \tan^2\theta$

$\sin 2\theta = \dfrac{1}{2}$

$2\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$\theta = \dfrac{\pi}{12}, \dfrac{5\pi}{12}$

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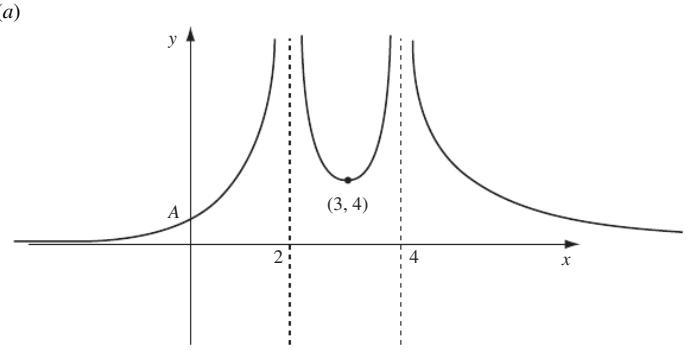
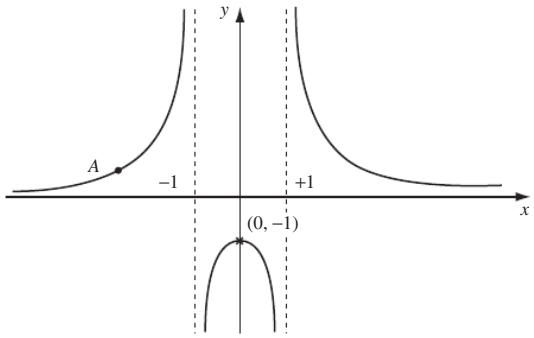
7. (*a*)



min. pt. at $(3, 4)$

pt. $A\left(0, \frac{1}{2}\right)$ **(5)**

(*b*)



asymptotes at $x = \pm 1$.

maximum at $(0, -1)$

$A(-3, 0.125)$ **(4)**

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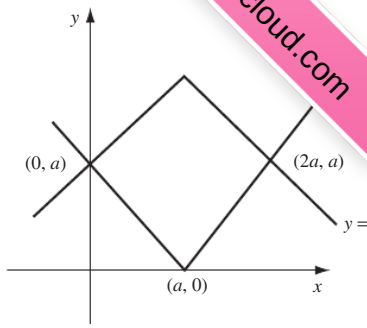
8. (*a*)



(*b*) points of intersection: $(0, a)$ and $(2a, a)$

(*c*) rhombus area $= \frac{1}{2}$ product of diagonals $= \frac{1}{2}(2a)($

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9. (*a*) $\cos\theta + 2\sin\theta = \sqrt{5}\left(\frac{1}{\sqrt{5}}\cos\theta + \frac{2}{\sqrt{5}}\sin\theta\right)$;

$R = \sqrt{5}$, $\tan\alpha = 2$

$R = 2.24$ (3 s.f.) $\alpha = 1.11^{c}$ (3 s.f.)

(*b*) $\left.\begin{matrix}\text{max.}\sqrt{5}\\ \text{min.}-\sqrt{5}\end{matrix}\right\}$, $\theta = 1.1$

(*c*) max. depth $15 + \sqrt{5}$ (17.24 metres 2 d.p.)

occurs at $\frac{\pi t}{12} = 1.11$ $\quad t = 4.24$ (2 d.p.)

(*d*) $t = 0$. $d = 15 + 1 = 16$ metres

(*e*) $d = 15 \Rightarrow \cos\left(\frac{\pi t}{12} - 1.11\right) = 0$

$\frac{\pi t}{12} - 1.11 = \frac{\pi}{2}$ $\quad t = 10.24$

time 2230 (nearest half hour)
